Computational Model of Vascular Biomechanics with Smooth Muscle Function

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***Abstract*— Understanding the brachial artery during functional dilation testing can allow physicians to noninvasively determine if the arterial wall has smooth muscle that has been damaged by disease. Since many other mechanical properties of the artery affect smooth muscle response a computer model would be useful to help interpret vascular smooth muscle function. In this article a biomechanical model of the human brachial artery was programmed using MATLAB to represent both the resting and vasodilated conditions of the brachial artery. The model results were evaluated in comparison with canine in vitro vessel data and human brachial artery intravascular ultrasound data. The model was found to accurately represent both vessels including the vasodilated and resting conditions.**

I. INTRODUCTION

One of the main causes of health concern in both men and women in the United States is heart disease. It is the cause of death in one of every four cases [1]. A source of heart disease is systemic hypertension which can be caused by arterial wall hypertrophy [2]. Systemic hypertension can lead to left ventricular hypertrophy, diastolic dysfunction, intracerebral hemorrhage due to medial degeneration, and accelerated atherosclerosis which is a cause of ischemic heart disease, stroke, and peripheral vascular disease [3]. Early diagnosis and treatment of systemic hypertension can decrease the health concern of heart disease. Vascular smooth muscle dysfunction is an early indication of vascular disease and damaged endothelium. The flow mediated dilation test with ultrasound imaging is a noninvasive measure of endothelial and vascular function. Unfortunately, the results of this test can be variable due to the multiple geometric and mechanical factors that affect dilation. We suggest that a computational biomechanical model may be useful to improve the interpretation of vascular dilation data.

Our model begins with the derivation of the arterial pressure-lumen area relationship. The pressure-area curve of a vessel incorporates most aspects of the vessel biomechanics in one data function for example, the pressure–area curve may be used to find the derivative of vessel area with respect to pressure [2].

II. METHODS

A. *Computational*

To generate the model for the brachial artery, MATLAB 2016b was utilized.

B. *Mechanical Modeling*

The functional relationship between vessel Pressure and lumen area was applied from Drzewiecki et al. who generated an equation from canine vascular measurements that describes the lumen area of the artery as it stretches and collapses [2]. As the vessel expands, the shape is more circular while its collapse generates a more elliptical shape. The original pressure equation is reproduced below:

$$Pt - a\left(e^{\frac{b(A-A_b)}{A_b}} - 1\right) - E\left(\left(\frac{A_b}{A}\right)^n - 1\right) + P_b \tag{1}$$

Where Pt is the transmural pressure, or the pressure difference between inside the artery and the external pressure, A_b and P_b are the area and pressure at vessel buckling (Pt=0), E is the elastic modulus of the vessel, and A is the area. Parameters a, b, and n are specific values for the equation to fit the data.

Equation (1) was solved so that area was in terms of pressure and then it was split into two, one for the stretch Eq. (2) and one for the collapse Eq. (3) of the artery. These solutions permit pressure to be the independent variable for better comparison with vascular data. The parameter set was chosen to create the complete area versus pressure curve.

$$A_{stretch} = A_b * \left(1 + \frac{1}{b} * \ln\left(\frac{Pt}{a} + 1\right)\right) \tag{2}$$

$$A_{collapse} = A_b * \left(1 + \frac{Pt - P_b}{E}\right)^{-1/n} \tag{3}$$

The nonlinear parameter estimation functions of MATLAB were then used to fit human intravascular ultrasound data from the area versus pressure curves from Banks et al. to obtain the values for P_b, A_b, E, a, b, and n for the resting and dilated states [4].

Compliance is defined as the derivative of volume with respect to pressure. In this study, the derivative was taken with area instead of volume so the units are mm^2/mmHg:

$$\frac{dA}{dP} = \frac{A(Pt + h) - A(Pt)}{h} \tag{4}$$

Further comparisons with the in vitro vessel data of Drzewiecki et al. and Banks et al. were made to ensure the model's accuracy [2,4].

III. Results

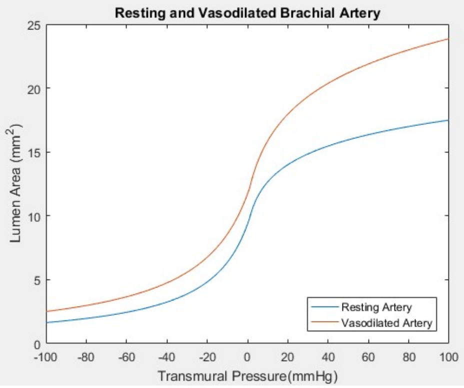
A. *Area versus Pressure for Rest and Dilation*

Data for the area versus transmural pressure curves obtained from Banks et al. was used to generate a best fit Eq. (2) and Eq. (3) from above [4]. Below are the generated values for both the resting and dilated models of the brachial artery. Next are the generated graph containing both models after Eq. (2) and Eq. (3) were pieced together.

TABLE I. Generated Constants

Model	A_B	P_B	E	a	b	n
Resting	9.0569 MM^2	-0.7 MMHG	22 DYNES/CM^2	2.667	3.9206	1
Dilated	11.4219 MM^2	-0.7 MMHG	28 DYNES/CM^2	5.1741	2.7657	1

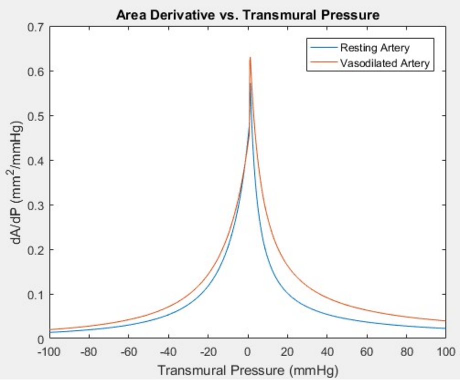
Fig. 1. Vessel area versus transmural pressure generated using best fit to invascular ultrsound human data [4].



B. *Vessel's Area Derivative*

To model the derivative, a step of h=0.25 was utilized.

Fig. 2. Brachial artery area derivative computed using Eq. (4).



C. *Comparison of Models*

For model verification, data points from the human brachial artery study that evaluated vessel stretch from Banks et al. was graphed on the pressure-area curve with the computational model for both resting and vasodilated vessel states [4]. For the derivative curve, only the resting vessel model was graphed with the canine data from Drzewiecki et al. for verification [2].

Fig. 3. Pressure-area curves from human data with computational model [4].

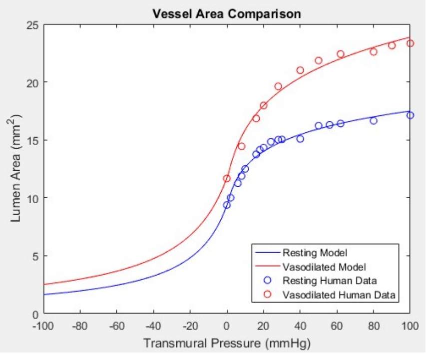
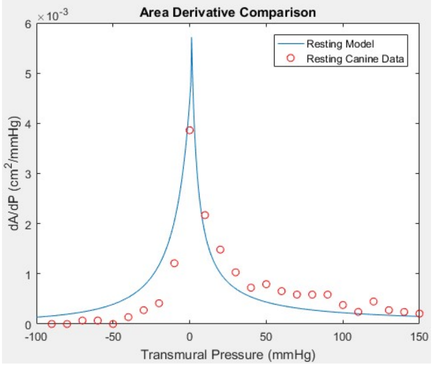


Fig. 4. Derivative curve from canine data with computational model [2].



Looking at the two graphs above, the compuational model closely represents the human data but the collapse of the vessel in the pressure-area curve doesn't reach zero at -100 mmHg. The derivative curve for a resting artery appears to be a shifted version of the canine data which is expected since canine physiology is different from that of humans.

IV. Conclusion

Describing the human brachial artery's collapse and expansion by using the mathematical model derived from the canine study is possible [2]. The equations fit the human data obtained from the invasive ultrasound study performed by Banks et al. [4]. Future work will include further model validation by examining the stress-strain curve of the artery and the change of elastic modulus with transmural pressure and smooth muscle function. Current model results have shown that it can successfully represent canine and human in vitro arteries intravascular measurements for both the resting and dilated conditions.

References

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